

**2014 NDIA GROUND VEHICLE SYSTEMS ENGINEERING AND TECHNOLOGY  
SYMPOSIUM  
MODELING & SIMULATION, TESTING AND VALIDATION (MSTV) TECHNICAL SESSION  
AUGUST 12-14, 2014 - NOVI, MICHIGAN**

**REDUCING STRUCTURAL WEIGHT AND INCREASING  
PROTECTION IN SIMPLE STRUCTURES  
SUBJECTED TO BLAST LOADS**

**Weiran Jiang**  
**Nickolas Vlahopoulos, PhD**  
NA&ME Department  
University of Michigan  
Ann Arbor, MI

**Matthew P. Castanier, PhD**  
**Ravi Thyagarajan, PhD**  
Analytics Department  
US Army TARDEC  
Warren, MI

**Syed Mohammad, PhD**  
Department of the Army Systems Coordinator  
Force Projection, Combat Sustainment Systems  
Crystal City, VA

**ABSTRACT**

*One of the main thrusts in current Army Science & Technology (S&T) activities is the development of occupant-centric vehicle structures that make the operation of the vehicle both comfortable and safe for the soldiers. Furthermore, a lighter weight vehicle structure is an enabling factor for faster transport, higher mobility, greater fuel conservation, higher payload, and a reduced ground footprint of supporting forces. Therefore, a key design challenge is to develop lightweight occupant-centric vehicle structures that can provide high levels of protection against explosive threats. In this paper, concepts for using materials, damping and other mechanisms to design structures with unique dynamic characteristics for mitigating blast loads are investigated. The Dynamic Response Index (DRI) metric [1] is employed as an occupant injury measure for determining the effectiveness of the each blast mitigation configuration that is considered. A model of the TARDEC Generic V-Hull structure comprises the vehicle considered in this study. The material properties and the configuration of the inner bulkheads that connect the outer V-shaped bottom with the inner floor are used as design parameters for reducing the DRI at a typical occupant location. In this particular example, it is demonstrated that the weight of the structure can be reduced by about ~12% and simultaneously, the DRI can be reduced by ~24%. This is achieved by creating an energy absorbing/decoupling mechanism between the outer hull, the inner floor, and the single degree of freedom upper torso system.*

**INTRODUCTION**

One of the main thrusts in current Army Science & Technology (S&T) activities is the development of occupant-centric vehicle structures that make the operation of the vehicle both comfortable and safe for the soldiers. Furthermore, a lighter weight vehicle structure is an enabling factor for faster transport, higher mobility, greater fuel conservation, higher payload, and a reduced ground footprint of supporting forces. Therefore, a key design challenge is to develop lightweight occupant-centric vehicle structures that can provide high levels of protection against explosive threats. Full system, end-to-end [8,13-16] as well as Reduced Order [17-19] Modeling and Simulation methodologies are extensively used for the development of blast-worthy ground vehicles in the Army Acquisition process.

The study presented in this paper investigates through simulation how the properties of materials used in the construction of a vehicle structure can alter the dynamic characteristics, offering improved isolation from the blast loads at a lower structural weight. A Generic V-Hull structure developed by TARDEC, *aka* the *TARDEC Generic Hull* [13] comprises the numerical model for investigating these concepts. The DRI, which is a standard occupant injury metric [1] for underbody simulations and testing, is used in this study as a measure of the structural performance with respect to Survivability. In the absence of an actual Anthropomorphic Test Device (ATD) in numerical models to measure lumbar loads, the DRI is the next best indicator of lumbar injury performance [10], and can be easily calculated from structural vehicle accelerations as shown in Appendix A.

In the literature, various concepts of employing the properties of materials as a mechanism to absorb energy have been presented. For example, utilizing shear thickening fluid due to its large capacity for energy absorption has been investigated [2-3, 9]. Shear thickening fluid is a specific type of non-Newtonian fluid with its viscosity dependent on the strain rate. It acts like a solid when experiencing a large shear load, such as an impulse of high pressure but of short duration from a blast, and returns to liquid form when the load is removed. Employing the plastic deformation induced in material for absorbing energy has been considered for designing blast-resistant structures [4]. The failure mechanisms in unidirectional fiber reinforced composites of delamination, fiber/matrix debonding, matrix cracking, and fiber breakage have been considered for creating blast mitigation configurations [5]. For similar purposes, functional graded metallic materials constructed in a layered sandwich configuration with several absorption layers have been also considered [6].

The concept of using properties of “softer” materials is investigated in this paper. It allows for higher deformation levels in the structure which in combination with a damping mechanism, can result in a reduced base excitation leading to lower DRIs and hence occupant injuries. Specifically, the properties of the inner bulkheads that connect the outer V-Hull bottom to the inner floor (Figure 3) are tuned in this manner, thereby offering an isolation mechanism that reduces the DRI metric. In the following sections of this paper, information is first presented about the numerical models employed in this study, namely the TARDEC V-Hull model and the DRI models. The software code LS-DYNA is used in the blast simulations and the viscoelastic material definition is used for defining the various properties of the internal bulkheads in the parametric study. Therefore, a brief discussion on the viscoelastic material definition in LS-DYNA is presented. Then, two different lumped parameter models for the DRI metric are described. In the first setup, a spring-mass model with a single degree of freedom (DOF) representing the upper torso of the occupant (Appendix A) is mounted directly in the middle of the inner floor. In the second setup, a three-DOF model representing the upper torso of the occupant, an energy-absorbing seat, and an energy-absorbing floor (Appendix B) is mounted to the hull. Finally, the process followed in the parametric study is discussed along with the final design configurations that reduce both the structural weight and the DRI metric.

## NUMERICAL MODELS

The TARDEC Generic V-Hull structure is presented in Figure 1. It comprises a representative but generic/notional vehicle structure that can be used in survivability research studies. The TARDEC Generic Hull experiment was designed to provide a notional geometry for underbody blast analysis and to evaluate blast mitigation technologies. Historically, the Department of

Army has had difficulty collaborating with industry and academia on underbody blast events due to the sensitive nature of the work. Data generated from testing military vehicles is typically sensitive and not readily sharable. To alleviate this issue, TARDEC has fabricated this generic vehicle hull with the intent to share data with academia and the industry to spur innovation in blast mitigation technologies. The main dimensions and the geometry of the V-Hull were used for creating a simplified model that is used in this work (Figure 2). The simplified model has the same thicknesses and material properties for the main structural components with the TARDEC V-Hull structure. It also contains inner bulkheads connecting the outer V-shaped bottom with the inner floor (Figure 3). The material properties of the bulkheads are used as design parameters in the parametric study. The airblast loading feature in LS-DYNA (\*LOAD\_BLAST) is used to represent a mine blast threat of TNT placed 0.2m below the bottom center of the vehicle.

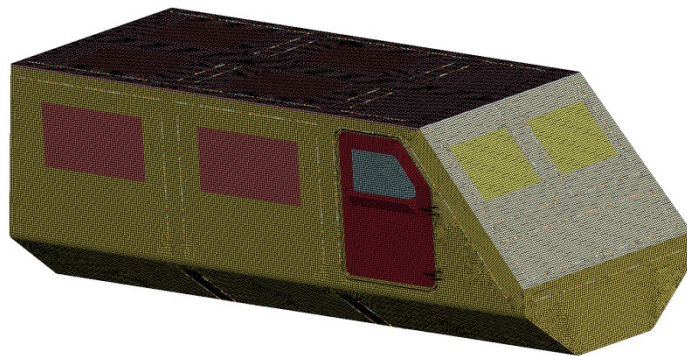


Figure 1. TARDEC V-Hull Structure

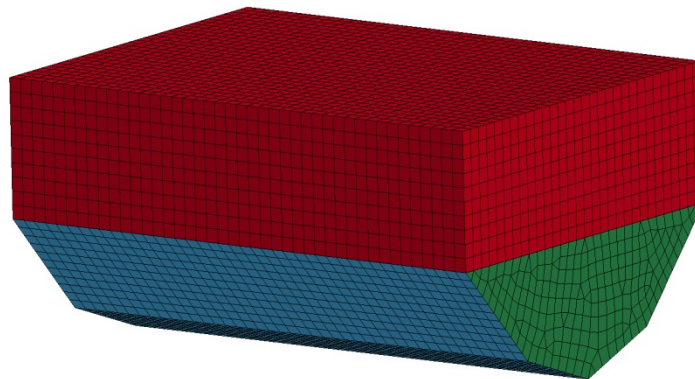


Figure 2. Simplified model of the TARDEC V-Hull structure that is used in this study

Figure 3a depicts the single-DOF (SDOF) lumped parameter model connected directly to the vehicle structure to evaluate the DRI. The upper part of the structure and the inner floor are removed from the figure in order for the internal bulkheads to be visible. These and all other parts that are omitted from the figure for visualization purposes are included in the simulations. The DRI is used as a metric for assessing the safety design characteristics of a vehicle. It represents the dynamic response of the lower lumbar region of an occupant. The DRI is

computed from the maximum dynamic compression measured in the spring, which is determined from the governing equations shown in Appendix A.

In the LS-DYNA model, a single spring-mass-damper system of mass 34.51 kg, with natural frequency equal to 52.9 rad/s and damping ratio of 0.224 is included in the finite element model of the simplified V-Hull. As mentioned earlier, a second three-DOF lumped parameter configuration is also considered, with the two intermediate DOFs representing the seat and the floor. This second configuration is presented in Figure 3b. In this case, the DRI is determined by the relative compression in the spring between the top DOFs. The nonlinear spring constants for the lowest (floor) and for the middle (seat) DOFs are shown in Figures 4a and 4b, respectively, and a damping coefficient of 9,220 (N\*sec)/m is used for the corresponding dampers. By including these one-DOF and three-DOF systems directly in the simulations, the calculations in the governing equations shown in Appendix A and B are automatically performed by LS-DYNA.

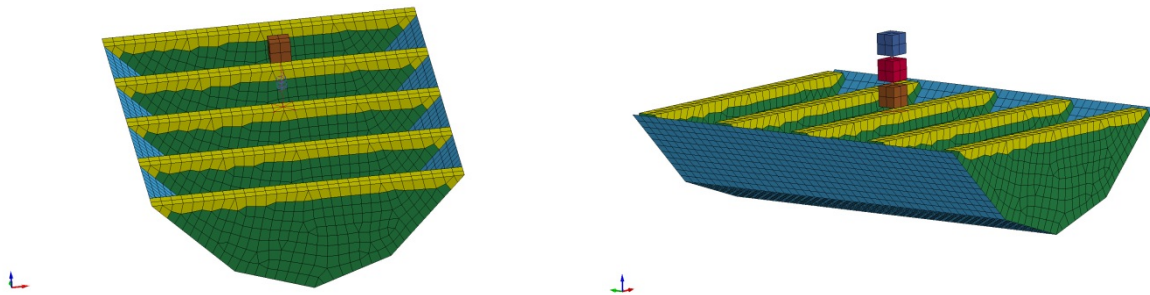


Figure 3. (a) SDOF model (left); (b) Three-DOF model (right) for determination of DRI

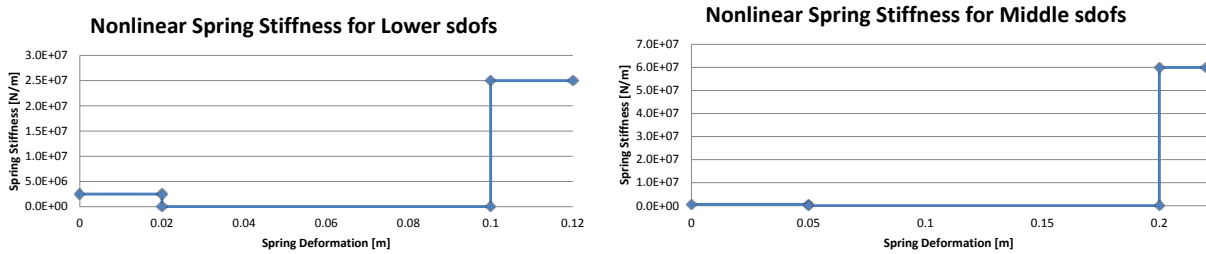


Figure 4. Spring stiffness curves for the floor (left) and seat (right) DOFs

The density, the modulus of elasticity and the dissipation properties of the material comprising the bulkheads are used in the parametric study for reducing simultaneously the weight of the structure and the DRI. The viscoelastic material definition of LS-DYNA (MAT\_061) is used for modeling this material [7], which models both viscous and elastic characteristics with a stress-strain relation that depends on the load history. It behaves as a spring-damper system and two classical models (Maxwell's and Kelvin's) are employed in the material definition. The parameters which are considered include: mass density, bulk modulus, short-time shear modulus, long-time shear modulus, and a decay constant. The bulk modulus, the short-time shear modulus, and the long-time shear modulus are determined by the instantaneous effective spring coefficient and the asymptotic effective spring coefficient. A linear relationship between the instantaneous effective spring coefficient  $E_0$  and the asymptotic effective spring coefficient  $E_\infty$  is used,  $E_0=1000 * E_\infty$ . Therefore, the Poisson's ratio of the material, the mass density, the decay constant, and the asymptotic spring coefficient are sufficient for defining the viscoelastic

Reducing Structural Weight and Increasing Protection in Simple Structures Subjected to Blast Loads, Jiang *et al*

UNCLASSIFIED: Distribution Statement A. Approved for public release (#25070)

material properties. In this study, these material properties are used for creating an isolation mechanism to reduce the occupant DRI at a reduced overall weight.

### PARAMETRIC STUDY

The configuration with the single-DOF model connected to the floor (Figure 3a) for evaluating the DRI was analyzed first. Initially, the parametric study attempted to change the density and the stiffness properties of the entire volume of each bulkhead part. Two main conclusions were drawn from this initial effort:

- First, it was decided to preserve the original steel properties for the upper part of each bulkhead (colored yellow in Figure 5) and alter the stiffness, the density, and the dissipation characteristics in the remaining portion of each bulkhead (colored green in Figure 5). The reason for this approach, is to avoid excessive local flexibility at the location where the SDOF model is attached to the floor when the bulkhead has reduced stiffness properties. The local flexibility at the attachment point makes it difficult to control the spring compression that determines the DRI.
- The second observation was that the overall mass of the vehicle has an impact to the overall rigid body response of the vehicle and thereby the DRI. Four equal lumped masses were added at the four corners of the vehicle to keep the total weight constant at the typical representative weight of such a vehicle. For each configuration, the values of the lumped masses were selected in a manner that the overall mass of the vehicle remained constant. This approach also reflects the ability to increase the payload of a vehicle even when the structure itself becomes lighter. The locations where the lumped masses were attached are shown in Figure 5.

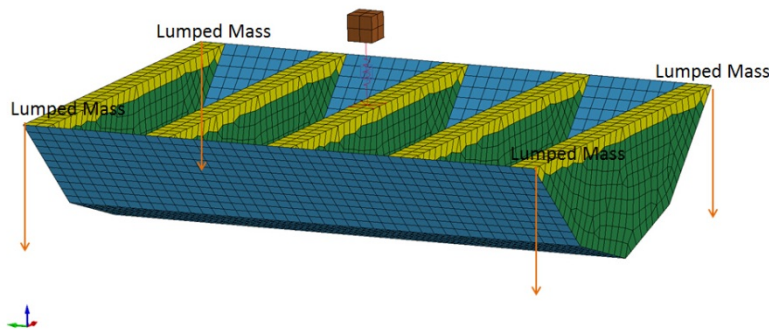


Figure 5. Partition of the bulkheads into two sections (yellow and green); and locations where lumped masses are attached for preserving the overall vehicle mass

The final configuration identified from the parametric study involving numerous configurations, had the following values: density equal to  $6,000\text{kg/m}^3$ , asymptotic spring coefficient equal to  $3.8 \times 10^6 \text{ N/m}$ , and a decay constant equal to 0.001. The Poisson's ratio did not vary and was set equal to 0.3. Figure 6 summarizes the time histories of deformation of the upper torso relative to the pelvis, for the original configuration (bulkheads made out of regular steel) and the final configuration. The values for the maximum spring compression and the associated DRI are also included in the Figure. An improvement of 24.2% is observed in the DRI while achieving a 12.5% reduction in the mass of the structure (as explained earlier the overall mass of the vehicle is retained constant for all configurations). Figures 7a and 7b present the displacement histories

for the attachment location on the floor where the SDOF is connected (location A), and for the upper torso DOF for the one-DOF lumped parameter DRI model (location B); the results are shown for the original and the final design configurations, respectively. Being total displacements, these results include the effects of both the flexible and the rigid body motion. It can be observed that the changes in the material properties of the bulkhead alter primarily the dynamic response of the connection point on the floor and in this manner reduce the maximum relative compression and the DRI.

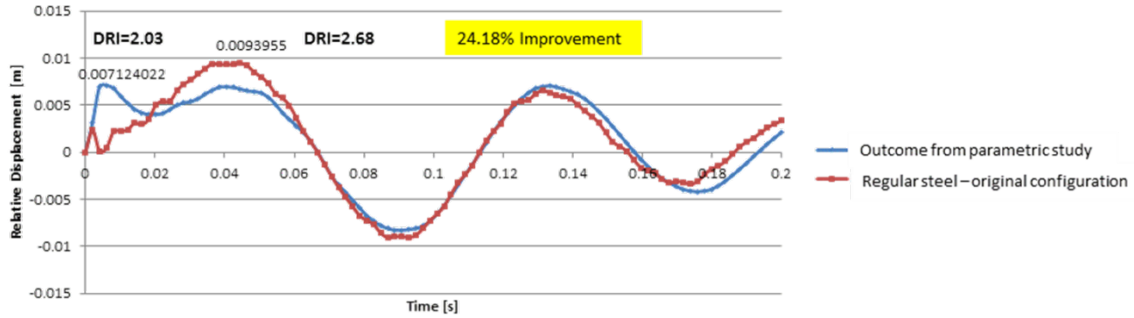


Figure 6. Time histories of deformation of the upper torso relative to the pelvis in the one-DOF lumped parameter DRI model

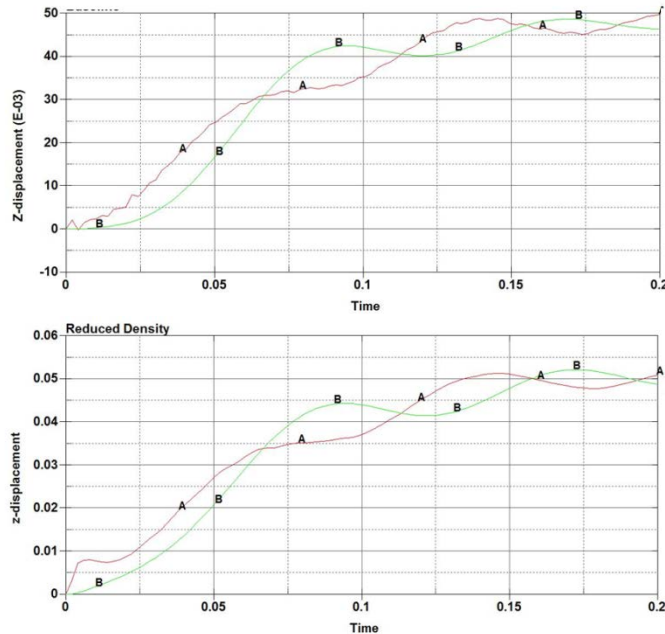


Figure 7. Dynamic displacements for (a) baseline configuration (top), (b) configuration resulting from parametric study (bottom). Point A is the pelvis (also floor attachment location), and B is the upper torso DOF of the one-DOF lumped parameter model.

In a similar manner, a parametric study was conducted, involving numerous configurations, for the three-DOF lumped parameter model for evaluating the DRI (Figure 3b). This parametric study retains the value of  $6,000 \text{ kg/m}^3$  for the lower section of each bulkhead, based on the configuration identified by the earlier work. The asymptotic stiffness and the decay constant comprise the varying parameters. For the final configuration, these two parameters acquire values of  $2 \cdot 10^6 \text{ N/m}$  and 0.005, respectively. Figure 8 summarizes the time histories of

deformation of the upper torso relative to the pelvis, for the original configuration (bulkheads made out of regular steel) and the final configuration. The values for the maximum spring compression and the associated DRI are also included in the figure. The improvement in the DRI is 34.2% this time, while the reduction in the mass of the structure remains at 12.5%. Figures 9a and 9b present the displacement histories for the attachment location on the hull where the three-DOF lumped mass system is connected (location A), and for the upper torso DOF for the three-DOF lumped parameter DRI model (location B), respectively. In this case, both the response of the upper torso DOF and the connection point are affected by the material properties of the bulkheads.

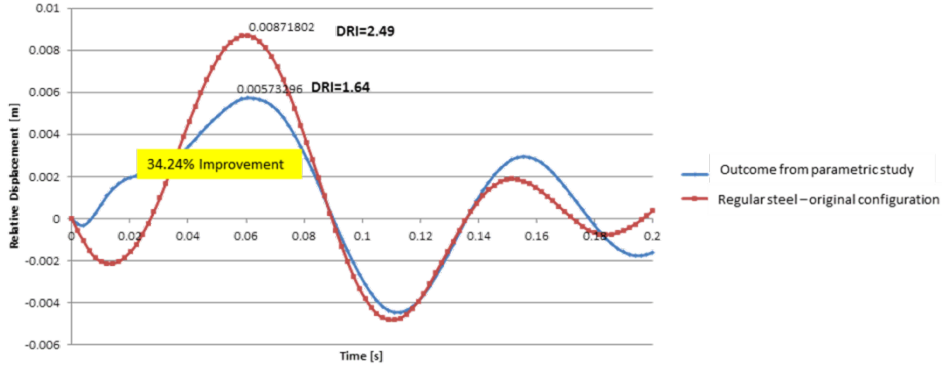


Figure 8. Time histories of deformation of the upper torso relative to the pelvis in the three-DOF lumped parameter DRI model

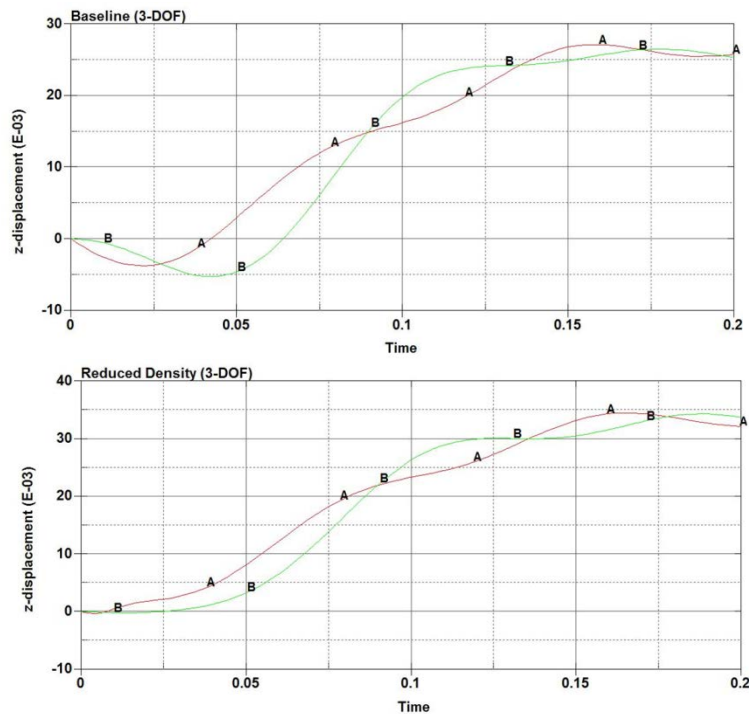


Figure 9. Dynamic displacements for (a) baseline configuration (top), (b) configuration resulting from parametric study (bottom). Point A is the pelvis, and B is the upper torso DOF of the three-DOF lumped parameter model

## CONCLUSIONS

This paper demonstrates how material properties can be used for tuning the dynamic behavior of a vehicle to simultaneously reduce the weight of the vehicle structure and occupant injuries. The intent is not to identify a specific material or design, but rather exercise a process for identifying suitable stiffness, inertia, and damping/absorptive characteristics of the various components. The results depend on how and where the seat is connected to the vehicle, the relative stiffnesses and energy absorption characteristics of stroking floors and seats [11], as well as the material properties. The selection process is driven by controlling and minimizing the energy that reaches the occupant from the blast and the resulting occupant injuries.

## ACRONYMS

ARC	Automotive Research Center
ATD	Anthropomorphic Test Device (Dummy)
CotS/COTS	Commercial-Off-the-Shelf
DA	Department of the Army
DoB/DOB	Depth of Burial
DoD/DOD	Department of Defense
DOF	Degree of Freedom
DRI	Dynamic Response Index
DTIC	Defense Technical Information Center <a href="http://www.dtic.mil">http://www.dtic.mil</a>
EA	Energy Absorption
FEA/FEM	Finite Element Analysis/Model
LS-DYNA	COTS structural dynamics software from Livermore Software Technology Corporation, CA
M&S	Modeling & Simulation
MDOF	Multiple Degree of Freedom
NA & ME	Naval Architecture and Mechanical Engineering
NATO	North Atlantic Treaty Organization
PM	Program Manager
R&D	Research & Development
RDECOM	Research, Development and Engineering Command
S&T	Science and Technology
SDOF	Single Degree of Freedom
STANAG	Standardization Agreement
TARDEC	Tank Automotive Research, Development and Engineering Center
UBB	UnderBody Blast
UBM	Underbody Blast Modeling/Methodology

## DISCLAIMER

Reference herein to any specific commercial company, product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the Dept. of the Army (DoA). The opinions of the authors expressed herein do not necessarily state or reflect those of the United States Government or the DoD, and shall not be used for advertising or product endorsement purposes.

## ACKNOWLEDGMENTS

The technical and financial support of the Automotive Research Center (ARC) in accordance with Cooperative Agreement W56HZV-04-2-0001 U.S. Army Tank Automotive Research, Development and Engineering Center (TARDEC) Warren, MI is acknowledged. Any opinions, findings and conclusions or recommendations in this paper are those of the author(s) and do not necessarily reflect the views of the U.S. Army TACOM Life Cycle Command.

Reducing Structural Weight and Increasing Protection in Simple Structures Subjected to Blast Loads, Jiang *et al*  
UNCLASSIFIED: Distribution Statement A. Approved for public release (#25070)

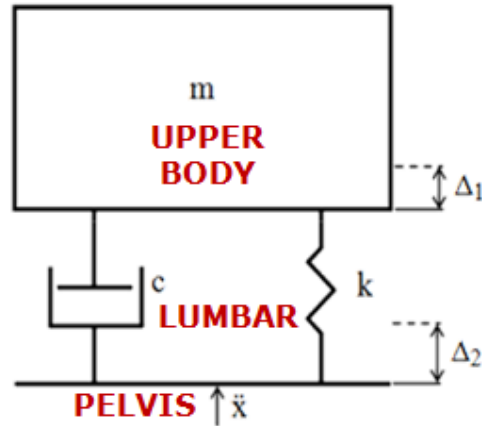


## REFERENCES

1. Final Report of HFM-090 Task Group 25, Test Methodology for Protection of Vehicle Occupants against Anti-Vehicular Landmine Effects, *NATO RTO TECHNICAL REPORT TR-HFM-090*, April 2007
2. XZ Zhang, WH Li, XL Gong. The rheology of shear thickening fluid (STF) and the dynamic performance of an STF-filled damper, *Smart Materials and Structures*, 2008.
3. KH Ahn, K Osaki. Mechanism of shear thickening investigated by a network model, *Journal of Non-Newtonian Fluid Mechanics*, 1995.
4. MS Hoo Fatt, H Surabhi. Blast resistance and energy absorption of foam-core cylindrical sandwich shells under external blast, *Composite Structures*, 2012.
5. RC Batra, NM Hassan. Blast resistance of unidirectional fiber reinforced composites, *Composites Part B: Engineering*, 2008
6. X Liu, X Tian, TJ Lu, D Zhou, B Liang. Blast resistance of sandwich-walled hollow cylinders with graded metallic foam cores, *Composite Structures*, 2012.
7. LS-DYNA Theory Manual, 06/25/14 (R5514), Livermore Software Technology Corporation, CA.
8. Thyagarajan, R., End-to-end System level M&S tool for Underbody Blast Events, *27th Army Science Conference, Army Technology Showcase*, Orlando, FL, Nov 29 – Dec 2. DTIC Report # ADA550921, TARDEC Registration # 21365, 2010.
9. YS Lee, ED Wetzell, NJ Wagner. The ballistic impact characteristics of Kevlar® woven fabrics impregnated with a colloidal shear thickening fluid, *Journal of materials science*, 2003.
10. Thyagarajan, R., Ramalingam, J., and Kulkarni, K.B., (2014), Comparing the Use of Dynamic Response Index (DRI) and Lumbar Load as Relevant Spinal Injury Metrics, *ARL Workshop on Numerical Analysis of Human and Surrogate Response to Accelerative Loading*, Jan 7-9, Aberdeen, MD, DTIC Report # AD A591409
11. Ramalingam, J. and Thyagarajan, R., Design and Analysis of Vertically Stroking Floors (OCP-TECD), *Internal Technical Briefing*, Jan 23, 2013
12. Manion, D., Computation and Validation of the Dynamic Response Index (DRI), DTIC Report # AD A591869, Aug 6, 2013
13. Daniel Dooge, Ramesh Dwarampudi, Grant Schaffner, Adam Miller, Ravi Thyagarajan, Madanmohan Vunnam & Venkatesh Babu (2011), Evolution of Occupant Survivability Simulation Framework Using FEM-SPH Coupling, *2011 NDIA Ground Vehicle Systems Engineering and Technology Symposium (GVSETS)*, Dearborn, MI, Aug 9-11. DTIC Report # AD A547566
14. Sun, J., Vlahopoulos, N., Stabryla, T., Goetz, R. et al., Blast Event Simulation for a Structure Subjected to a Landmine Explosion, SAE Technical Paper 2006-01-0931, 2006
15. Vlahopoulos, N. Zhang, G, Validation of a Simulation Process for Assessing the Response of a Vehicle and Its Occupants to an Explosive Threat, *27th Army Science Conference*, Orlando, FL, 2010
16. Cummins, C., Modeling brown clayey sand in LS-DYNA with an analytical method for soil state initialization, DTIC Report # AD B398007, FOUO/Distribution C. ARL Report # ARL-TR-6830. Requests for this document shall be referred to Director, U.S. Army Research Laboratory, ATTN: RDRL-WMP-F, Aberdeen Proving Ground, MD 21005-5066, February 2014
17. Kulkarni, K., Ramalingam, J., and Thyagarajan, R. (2014), Assessment of the Accuracy of Certain Reduced Order Models used in the Prediction of Occupant Injury during Under-Body Blast Events, *SAE Int. J. Trans. Safety* 2(2):2014, doi:10.4271/2014-01-0752, DTIC Report # AD A599846

18. Kulkarni, K.B., Ramalingam, J., and Thyagarajan, R. (2013), Evaluating the Effectiveness of Various Blast Loading Descriptors as Occupant Injury Predictors for Underbody Blast Events, NDIA Ground Vehicle Systems Engineering and Technology Symposium (GVSETS) Modeling & Simulation, Testing and Validation (MSTV) Mini-Symposium August 21-22, Troy, Michigan. DTIC Report # AD A590537
19. Li, L., Stowe, N., Vlahopoulos, N., Mohammad, S., Barker, C., Thyagarajan, R. (2013), Utilization of Fast Running Models in Buried Blast Simulations of Ground Vehicles for Significant Computational Efficiency, NDIA Ground Vehicle Systems Engineering and Technology Symposium (GVSETS) Modeling & Simulation, Testing and Validation (MSTV) Mini-Symposium August 21-22, Troy, Michigan, DTIC Report # AD A590114
20. Jaisankar Ramalingam, Sherri Chandra, Ravi Thyagarajan (2013), Reduced Order Modeling for Rapid Simulations of Blast and Rollover Events of a Ground Vehicle and its Occupants Using Rigid Body Dynamic Models, DTIC Report # AD A585048
21. NATO AEP-55 STANAG 4569, NATO Standardization Agreement covering the standards for the Protection Levels for Occupants of Logistic and Light Armored Vehicles, [http://en.wikipedia.org/wiki/STANAG\\_4569](http://en.wikipedia.org/wiki/STANAG_4569)

**Appendix A: Dynamic Response Index (DRI) – SDOF Mechanical Model [10]**

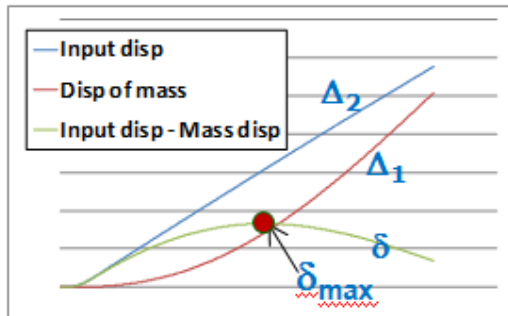
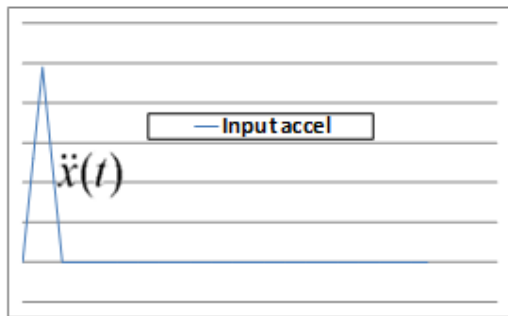


$$\ddot{x}(t) = \ddot{\delta} + 2\xi\omega_n\dot{\delta} + \omega_n^2\delta$$

- $\delta$  is the relative displacement between the upper body and pelvis ( $\delta = \Delta_1 - \Delta_2$ )
- $\zeta$  is the damping coefficient<sup>33</sup> (0.224)  $\zeta = c / (2 * \sqrt{m * k})$
- $\omega_n$  is the natural frequency<sup>33</sup> (52.9 rad/s)  $\omega_n = \sqrt{k/m}$
- **Normalized Lumbar Force**

$$= (k * \delta_{max}) / (mg) = \omega_n^2 * \delta_{max} / g$$

$$DRI = \frac{\omega_n^2 \delta_{max}}{g}$$

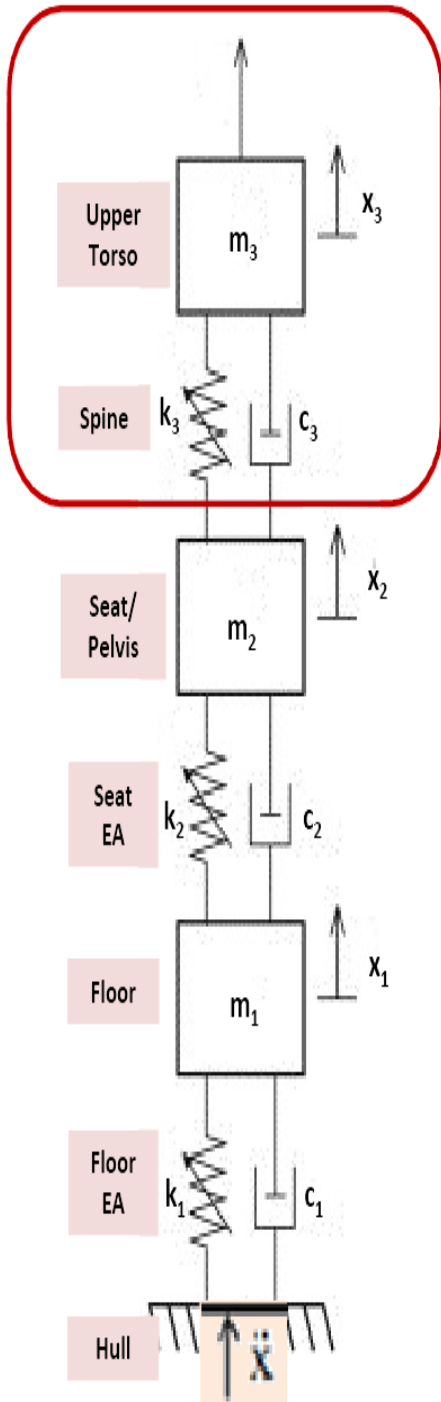


**m = 34.51 kg**  
**k = 9.66E04 N/m**  
**c = 818.1 Nsec/m**

**therefore:**

**$\omega_n = 52.9$  rad/s**  
 **$\zeta = 0.224$**

**Appendix B: Dynamic Response Index (DRI) – Three-DOF Mechanical Model [11, 12]**



- Takes hull acceleration as input.
- Accounts for energy absorption by the floor and seat.
- Springs representing the floor and seat are piecewise-linear.
- The spring representing the spine is still linear.

$$m_3 \ddot{x}_3(t) = F_{spring3} + F_{damper3} \quad z_3 = x_3 - x_2$$

$$m_3 \ddot{z}_3(t) = -k_3(x_3 - x_2) - c_3(\dot{x}_3 - \dot{x}_2) \quad z_2 = x_2 - x_1$$

$$m_3(\ddot{z}_3 + \ddot{x}_2) = -k_3 z_3 - c_3 \dot{z}_3 \quad z_1 = x_1 - x$$

$$m_3 \ddot{z}_3(t) = -m_3 \ddot{x}_2 - k_3 z_3 - c_3 \dot{z}_3$$

$$m_2 \ddot{x}_2(t) = -F_{spring3} - F_{damper3} + F_{spring2} + F_{damper2}$$

$$m_2 \ddot{x}_2(t) = k_3(x_3 - x_2) + c_3(\dot{x}_3 - \dot{x}_2) - k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1)$$

$$m_2(\ddot{z}_2 + \ddot{x}_1) = k_3 z_3 + c_3 \dot{z}_3 - k_2 z_2 - c_2 \dot{z}_2$$

$$m_2 \ddot{z}_2(t) = -m_2 \ddot{x}_1 + k_3 z_3 + c_3 \dot{z}_3 - k_2 z_2 - c_2 \dot{z}_2$$

$$m_1 \ddot{x}_1(t) = -F_{spring3} - F_{damper3} + F_{spring2} + F_{damper2}$$

$$m_1 \ddot{x}_1(t) = k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) - k_1(x_1 - x) - c_1(\dot{x}_1 - \dot{x})$$

$$m_1(\ddot{z}_1 + \ddot{x}) = k_2 z_2 + c_2 \dot{z}_2 - k_1 z_1 - c_1 \dot{z}_1$$

$$m_1 \ddot{z}_1(t) = -m_1 \ddot{x} + k_2 z_2 + c_2 \dot{z}_2 - k_1 z_1 - c_1 \dot{z}_1$$